Supplementary Material

There is one integral which needs to be evaluated in order to get the analytical expression for the lesser self-energy. Here we write t instead of $t-t'$ for brevity.

$$I^< = \int_{-\infty}^{\infty} d\epsilon\, f(\epsilon-\mu)e^{-i\epsilon t} = \int_{-\infty}^{\infty} d\epsilon\, \frac{1}{1+e^{\beta(\epsilon-\mu)}}e^{-i\epsilon t} = \int_{-\infty}^{\infty} d\epsilon\, \frac{1}{1+e^{\beta(\epsilon-\mu)}}e^{-i\epsilon t}e^{\eta\epsilon}$$

Close contour in lower half plane since $t<0$, write $\tilde{t} = t + i\eta$. The poles are at $\beta(\epsilon_n - \mu) = -i\pi - 2\pi in$ with $n \in \mathbb{Z}$. If one has a finite hopping in the leads you get roughly $t_R\beta/\pi$ poles, since the integral has bounds $\pm 2t_R$ and β makes the poles denser. This means that the sum below has a finite amount of terms and can therefore be well approximated by a finite amount of exponentials.

$$\begin{aligned} I^< &= -2\pi i \sum_{n=0}^{\infty} e^{-i\epsilon_n\tilde{t}}\mathrm{Res}\left[\frac{1}{1+e^{\beta(\epsilon_n-\mu)}}\right] = 2\pi i/\beta \sum_{n=0}^{\infty} e^{-(\pi+i\mu\beta+2\pi n)\tilde{t}/\beta} \\ &= 2\pi i/\beta e^{-\pi\tilde{t}/\beta - i\mu\tilde{t}} \sum_{n=0}^{\infty} e^{-2\pi n\tilde{t}/\beta} = 2\pi i/\beta e^{-\pi\tilde{t}/\beta - i\mu\tilde{t}} \frac{1}{1-e^{-2\pi\tilde{t}/\beta}} = e^{-i\mu\tilde{t}}\frac{i\pi/\beta}{\sinh\big(\pi\tilde{t}/\beta\big)} \end{aligned}$$

$$\begin{aligned} \mathrm{Res}\left[\frac{1}{1+e^{\beta(\epsilon_n-\mu)}}\right] &= \lim_{\epsilon\to i/\beta(\pi+2\pi n)+\mu} \frac{(\epsilon-(\pi+2\pi n)i/\beta+\mu)}{1+e^{\beta(\epsilon-\mu)}} = \lim_{\epsilon\to i/\beta(\pi+2\pi n)} \frac{(\epsilon-(\pi+2\pi n)i/\beta)}{1+e^{\beta\epsilon}} \\ &= \lim_{x\to 0}\frac{i/\beta x}{1+e^{i(x+\pi)}} = \lim_{x\to 0}\frac{i/\beta x}{1-(1+(ix)+\mathcal{O}((ix)^2))} = -1/\beta \end{aligned}$$

To generalize this result we consider the behavior for t close to 0.

$$e^{-i\mu\tilde{t}}\frac{i\pi/\beta}{\sinh\big(\pi\tilde{t}/\beta\big)} = e^{-i\mu\tilde{t}}\frac{i\pi/\beta}{\pi\tilde{t}/\beta + \mathcal{O}\big((\pi\tilde{t}/\beta)^2\big)} \to \mathcal{P}\left\{\frac{ie^{-i\mu t}}{t}\right\} + \pi\delta(t) \tag{S1}$$

Hence

$$I^< = \mathcal{P}\left\{e^{-i\mu t}\frac{i\pi/\beta}{\sinh(\pi t/\beta)}\right\} + \pi\delta(t)$$

Since this is an integral coming from the lesser self-energy it is multiplied by $\theta(t)$, which means that in total we have:

$$\mathcal{P}\left\{e^{-i\mu t}\frac{i\pi/\beta}{\sinh(\pi t/\beta)}\right\} + \delta(t)\pi/2$$

This result has been obtained earlier in [*Phys. Rev. B* **95**, 165440 (2017)].The low temperature limit is:

$$\mathcal{P}\left\{\frac{ie^{-i\mu t}}{t}\right\} + \delta(t)\pi/2 \qquad \text{when } \beta\to\infty$$

$$\int_{-\infty}^{\infty} d\epsilon\,\theta(t)e^{-i\epsilon t} = \int_{-\infty}^{\infty} d\epsilon\Big(-\frac{1}{2\pi i}\int_{-\infty}^{\infty} d\epsilon'\,\frac{1}{\epsilon'+i\eta}\Big)e^{-i\epsilon' t}e^{-i\epsilon t} = -\frac{1}{2\pi i}\int_{-\infty}^{\infty} d\epsilon\int_{-\infty}^{\infty} d\epsilon'\,e^{-i\epsilon t}\frac{1}{\epsilon'+i\eta}e^{-i\epsilon' t}$$
$$= -\frac{1}{2\pi i}\int_{-\infty}^{\infty} d\epsilon\Big(-i\pi e^{-i\epsilon t} + \mathcal{P}\int_{-\infty}^{\infty} d\epsilon'\Big(\frac{1}{\epsilon'}\Big)e^{-i(\epsilon'+\epsilon)t}\Big)$$
$$= \frac{1}{2}\int_{-\infty}^{\infty} d\epsilon e^{i\epsilon t} - \frac{1}{2\pi i}\int_{-\infty}^{\infty} d\epsilon\,\mathcal{P}\Big\{\int_{-\infty}^{\infty} dx\frac{e^{-ixt}}{x-\epsilon}\Big\} = \pi\delta(t)$$

The second integral is the Hilbert transform e^{-ixt} in x, if $t \neq 0$ then the Hilbert transform is $i\mathrm{sgn}(t)e^{-i\epsilon t}$ which gives $\sim \delta(t)$ when integrated with ϵ and therefore it is zero. If $t = 0$ we have the Hilbert transform of a constant, hence it's again zero.
The WBL matrix is given by

$$[\Gamma^\alpha]^{\sigma\sigma'}_{ij} = \gamma\delta_{\alpha_c i_x}\delta_{\alpha_c j_x}\frac{\delta_{\alpha_s\sigma'}\delta_{\alpha_s\sigma}}{\pi}\sum_{k_y}\frac{4}{N_y+1}\sin\left(\frac{\pi k_y i_y}{N_y+1}\right)\times\sin\left(\frac{\pi k_y j_y}{N_y+1}\right) \tag{S2}$$

The alpha label contains spin $\alpha_s =\uparrow,\downarrow$ and coordinate $\alpha_c = 1, N$ indices. N is the rightmost site.
In the main text it is claimed that the following integral gives a time-linear evolution:

$$\int_{t_0}^{t} \mathrm{d}\bar{t}\sum_\alpha e^{-i\int_t^{\bar{t}} V_\alpha(\tilde{t})\mathrm{d}\tilde{t}}\Lambda_\alpha(t-\bar{t})s(t)s(\bar{t})G^0(t-\bar{t}) \tag{S3}$$
$$= s(t)\sum_\alpha\int_{t_0}^{t} \mathrm{d}\bar{t}e^{-i\int_t^{\bar{t}} V_\alpha(\tilde{t})\mathrm{d}\tilde{t}}\Lambda_\alpha(t-\bar{t})s(\bar{t})G^0(t-\bar{t}) = s(t)\sum_\alpha I_\alpha(t) \tag{S4}$$

$$I_\alpha(t) = \int_{t_0}^{\tau} \mathrm{d}\bar{t}e^{-i\int_t^{\bar{t}} V_\alpha(\tilde{t})\mathrm{d}\tilde{t}}\Lambda_\alpha(t-\bar{t})s(\bar{t})G^0(t-\bar{t}) + \int_{\tau}^{t} \mathrm{d}\bar{t}e^{-i\int_t^{\bar{t}} V_\alpha(\tilde{t})\mathrm{d}\tilde{t}}\Lambda_\alpha(t-\bar{t})s(\bar{t})G^0(t-\bar{t}) \tag{S5}$$
$$= \int_{t_0}^{\tau} \mathrm{d}\bar{t}e^{-iV_\alpha(t-\bar{t})}\Lambda_\alpha(t-\bar{t})\sin(\pi\bar{t}/2\tau)^2G^0(t-\bar{t}) + \int_{\tau}^{t} \mathrm{d}\bar{t}e^{-iV_\alpha(t-\bar{t})}\Lambda_\alpha(t-\bar{t})G^0(t-\bar{t}) \tag{S6}$$
$$= I_{\alpha,1}(t) + I_{\alpha,2}(t) \tag{S7}$$

$$I_{\alpha,1} = \int_{t_0}^{\tau} \mathrm{d}\bar{t}e^{-iV_\alpha(t-\bar{t})}\Lambda_\alpha(t-\bar{t})\Big(\frac{1}{2} - \frac{1}{4}e^{-i\pi\bar{t}/\tau} - \frac{1}{4}e^{i\pi\bar{t}/\tau}\Big)G^0(t-\bar{t}) = \frac{1}{2}I^0_{\alpha,1} - \frac{1}{4}I^+_{\alpha,1} - \frac{1}{4}I^-_{\alpha,1} \tag{S8}$$

$$I_{\alpha,1}^{\Omega}(t+\Delta) = \int_{t_0}^{\tau} \mathrm{d}\bar{t} e^{-iV_\alpha(t+\Delta-\bar{t})}\Lambda_\alpha(t+\Delta-\bar{t})e^{i\Omega\pi\bar{t}/\tau}G^0(t+\Delta-\bar{t}) \tag{S9}$$

$$= \int_{t_0-\Delta}^{\tau-\Delta} \mathrm{d}y e^{-iV_\alpha(t-y)}\Lambda_\alpha(t-y)e^{i\Omega\pi(y+\Delta)/\tau}G^0(t-y) \tag{S10}$$

$$= \left(\int_{t_0-\Delta}^{t_0} + \int_{t_0}^{\tau} - \int_{\tau-\Delta}^{\tau}\right)\mathrm{d}y\, e^{-iV_\alpha(t-y)}\Lambda_\alpha(t-y)e^{i\Omega\pi(y+\Delta)/\tau}G^0(t-y) \tag{S11}$$

$$= e^{i\Omega\Delta/\tau}I_{\alpha,1}^{\Omega}(t) + \left(\int_{t_0-\Delta}^{t_0} - \int_{\tau-\Delta}^{\tau}\right)\mathrm{d}y\, e^{-iV_\alpha(t-y)}\Lambda_\alpha(t-y)e^{i\Omega\pi(y+\Delta)/\tau}G^0(t-y) \tag{S12}$$

$$\approx e^{i\Omega\Delta/\tau}I_{\alpha,1}^{\Omega}(t) + \frac{\Delta}{2}\Bigg[e^{-iV_\alpha(t-t_0+\Delta)}\Lambda_\alpha(t-t_0+\Delta)e^{i\Omega\pi(t_0)/\tau}G^0(t-t_0+\Delta) \tag{S13}$$

$$+ e^{-iV_\alpha(t-t_0)}\Lambda_\alpha(t-t_0)e^{i\Omega\pi(t_0+\Delta)/\tau}G^0(t-t_0)\Bigg] \tag{S14}$$

$$- \frac{\Delta}{2}\Bigg[e^{-iV_\alpha(t-\tau+\Delta)}\Lambda_\alpha(t-\tau+\Delta)e^{i\Omega\pi(\tau)/\tau}G^0(t-\tau+\Delta)) \tag{S15}$$

$$+ e^{-iV_\alpha(t-\tau)}\Lambda_\alpha(t-\tau)e^{i\Omega\pi(\Delta+\tau)/\tau}G^0(t-\tau)\Bigg] \tag{S16}$$

So we need to evolve two GF:s, $G^0(t-t_0)$ and $G^0(t-\tau)$. The second one is only needed when $t > \tau$, since before then this integral is up to $t+\Delta$ and the change of variables on gives one new interval instead of two.

$$I_{\alpha,2}(t+\Delta) = \int_{\tau}^{t+\Delta} \mathrm{d}\bar{t} e^{-iV_\alpha(t+\Delta-\bar{t})}\Lambda_\alpha(t+\Delta-\bar{t})G^0(t+\Delta-\bar{t}) \tag{S17}$$

$$= \int_{\tau-\Delta}^{t} \mathrm{d}y e^{-iV_\alpha(t-y)}\Lambda_\alpha(t-y)G^0(t-y) \tag{S18}$$

$$= I_{\alpha,2}(t) + \int_{\tau-\Delta}^{\tau} \mathrm{d}y e^{-iV_\alpha(t-y)}\Lambda_\alpha(t-y)G^0(t-y) \tag{S19}$$

$$\approx I_{\alpha,2}(t) + \frac{\Delta}{2}\Bigg[e^{-iV_\alpha(t-\tau+\Delta)}\Lambda_\alpha(t-\tau+\Delta)G^0(t-\tau+\Delta) \tag{S20}$$

$$+ e^{-iV_\alpha(t-\tau)}\Lambda_\alpha(t-\tau)G^0(t-\tau)\Bigg] \tag{S21}$$

This integral requires the GF $G^0(t-\tau)$ which we are already computing.